How Well Can Students Determine Simple Growth and Decay Patterns From a Diagram?

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Abstract

Six of the 35 national (United States) mathematics standards for high school (ages 15 to 18) instruction relate very well to the use of system dynamics modeling methods, especially in algebra. A very important concept, recognizing the underlying factors that produce different patterns of behavior could be considered a unifying theme throughout algebra, pre-calculus, and calculus. Most of the assignments given to algebra students do not ask students to consider this underlying theme (growth/decay pattern) except when studying a particular function. They never combine functions they have studied in isolation to determine potentially new growth or decay patterns. An assessment was given to 62 advanced algebra students to determine if they could predict the growth or decay patter for six different scenarios, given a STELLA diagram and a few numbers. This paper explains the assessment given and presents the results produced by the 62 students.

When reading the US national mathematics standards for high school instruction there are six standards that the system dynamics method seems to directly address. Those standards indicated that the student should be able to:

1. use mathematical models to represent and understand quantitative relationships,
2. analyze (rates of) change in various contexts,
3. apply and adapt a variety of appropriate strategies to solve problems,
4. understand how mathematical ideas interconnect and build on one another to produce a coherent whole,
5. recognize and apply mathematics in contexts outside of mathematics, and
6. use representations to model and interpret physical, social and mathematical phenomena.

In the Vision Statement for the national mathematics standards it states that we want students to "draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find the methods that enable them to make progress." The national standards document goes on to state that "technology not only influences how mathematics is taught and learned but also affects what is taught and when a topic appears in the curriculum." Both of these statements also point to the use of system dynamics modeling as an appropriate and important problem solving strategy in high school mathematics courses, especially algebra, pre-calculus, and calculus, where it would most apply.

These standards provide guidance, but there are also unifying themes that transcend the different functions taught in algebra. In fact, the unifying theme comes from the reform calculus movement, started over twenty years ago in the United States to improve student understanding of mathematics. That theme is determining the type of behavior a particular problem will produce. Once a student reads a story problem they need to think about the type of behavior that the problem describes. Is the behavior linear, exponential, convergent, logarithmic, parabolic, or sinusoidal (among other possibilities). Once they have chosen the type of function, they need to determine whether the problem suggests that the model should produce growth or decay (because traditionally we never mix the two within the same problem). Then the student defines his mathematical equation, hoping that the behavior of the equation will produce the desired pattern.

Due to the level of abstraction required for students to model linear and exponential phenomenon using equations, they are never required to combine both rates of growth and rates of decay in one problem, nor combine linear and exponential function behavior in one problem. This places a serious limitation on the types of problems students at this level can study. Additionally it limits any serious discussion of feedback in any of the story scenarios. If it is the goal of our society to improve our young students' ability to analyze more realistic behavior patterns that occur in the word, so they might have an opportunity to help develop some intuition about the cause and possible modification of behavior patterns, it becomes necessary to make available other modeling tools and techniques that will allow our students a chance to analyze slightly more complicated behavior patterns, ones that are modest extensions of patterns they should know well.

This author has used STELLA diagrams, as well as equations, in algebra classes to represent mathematical models for various story problems studied in class. Observation and experimentation have indicated that the diagrams are easier for students to understand, especially if the story contains multiple functions. So, the diagram representation was used to determine if 62 advanced algebra (second year algebra) students could determine, from the diagram, the description of each icon, and a few numbers provided, whether the model would produce growth or decay behavior. The diagram was also used to determine if the student could then identify the specific type of pattern for the growth or decay behavior. (In hindsight, it would have been better to have students work with both the equation representation and the diagram representation for a story problem, to gather further data to determine if the diagram was really easier to read.)

The Preparation

Students in the two advanced algebra classes ranged from grades 8 (14 years) to grade 12 (17 years). The students in these classes used the STELLA software in the first semester of the year (2007-08) on the following occasions. When studying linear functions, they spent one class period (50 to 60 minutes) in the computer lab, building simple linear models for three problems. When studying exponential functions another class period was devoted to building exponential growth or exponential decay models. Similarly, one class period was spent building quadratic models for various problems when the class was studying quadratic functions. Near the end of the semester (January 2008) the classes spent three days building and analyzing very simple drug models (a combination of linear and exponential structures). On the fourth day, at the end of the first semester the students were given a problem involving population growth and food production. This experiment was written up as "The Malthus Problem" last year for the ISDC 2008. On the fifth day students were given another exercise. The fifth exercise asked students to identify whether a given scenario would exhibit growth or decay and were asked to select one of three possible growth patterns or one of three possible decay patterns that the scenario should produce. They were also asked if there was any feedback evident in the problem, and, if so, whether it was reinforcing or balancing feedback.

The Exercise

Nowhere in high school mathematics instruction in the United States is there ever a discussion of feedback analysis for any story scenarios presented in class. To help set the stage for a simple introduction of reinforcing and balancing feedback, used in the main exercise, a preliminary exercise was distributed.

The preliminary exercise

Students were given four scenarios and were asked to explain how the two 'actors' or components might interact with one another. The first scenario involved an ecosystem containing wolves and rabbits. The wolf population is suddenly increased by a few wolves in the second month, and students are asked to draw how they think the wolf and the rabbit populations will change over the course of one year. They were then to explain why they drew the graphs as they did. The second scenario involved a student's practice time either playing a musical instrument or an athletic sport, and his/her subsequent performance playing the musical instrument or playing the sport, over a period of one year. Again graphs and explanations were required. The third scenario involved the connection between births in a city and the population of the city, over a period of one year. And finally, the connection between hunger and eating over the period of one day was to be graphed and explained.[1]

The Main Exercise

At the beginning of the exercise a short definition of reinforcing feedback, and a short definition of balancing feedback was given. The students were asked to determine what type of feedback was present in the wolf-rabbit relationship, and which type was present in the births-population relationship given in the preliminary exercise. Then students were given the following six scenarios to analyze.

Students were to read the brief description of the scenario and look at the STELLA diagram that was displayed to model the scenario. Then they were to decide if the model would exhibit growth or decay behavior. For each type of behavior there were three patterns of either growth

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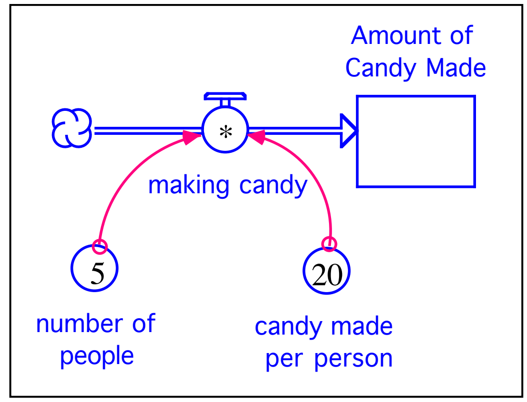
[1] Each of these scenarios was extracted from the paper by Linda Booth-Sweeney and John Sterman, Thinking about systems: student and teacher conceptions of natural and social systems. *System Dynamics Review 23*.

or decay, from which one pattern was to be selected. Finally, the students were to determine if any feedback was present in the model. If there was feedback, they were to identify whether it was positive or negative, and were to explain why they thought it was of that type.

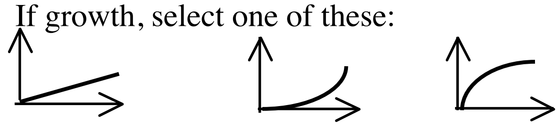
Scenario 1: Making Candy[2]



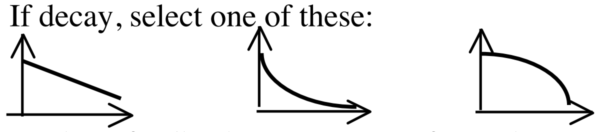
a. Will the amount of candy (shown in the diagram at the right) grow or decay over time? ______________

b. What will the pattern of growth or decay look like. Circle one graph below.

If growth, select one of these:



If decay, select one of these:



c. Is there feedback? _________ If so, what type of feedback is it? ______________ Explain

The scenarios started out very simply, displaying linear growth, then exponential growth, both of which students should have been very familiar with from previous class exercises. Then the structures were combined, the third being a population structure with both birth and death (exponential) flows. The fourth and fifth combined both a linear and exponential flow, one as inflow and the other as outflow. The linear inflow/exponential outflow students had experience with in the drug model exercises completed a few days before this exercise. But the exponential inflow and linear outflow students had not experienced before. The final structure, a population model structure with a non-renewable food resource upon which it depended, was a much more difficult scenario, but given just to see if any students could analyze it.

[2] See the other five scenarios in appendix A of this paper. Briefly, scenario 2 shows exponential inflow, no outflow. Scenario 3 shows exponential inflow and outflow, growth fraction smaller than decay fraction. Scenario 4 shows linear inflow, exponential outflow. Scenario 5 shows exponential inflow, linear outflow. Scenario 6 shows population model with exponential inflow and outflow, but decay rate depends upon a non-renewable resource.

The Results:

A graphical representation of the results appear below. The actual data is displayed in appendix B.

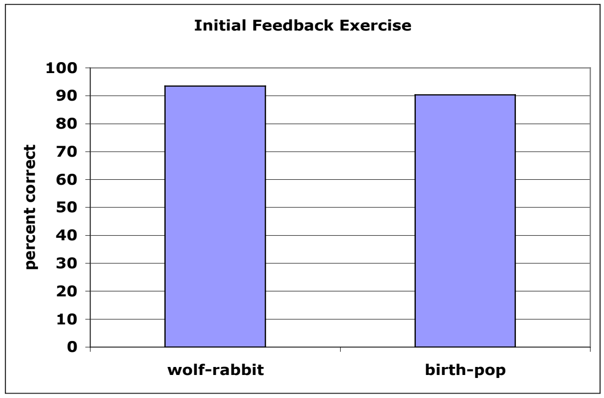
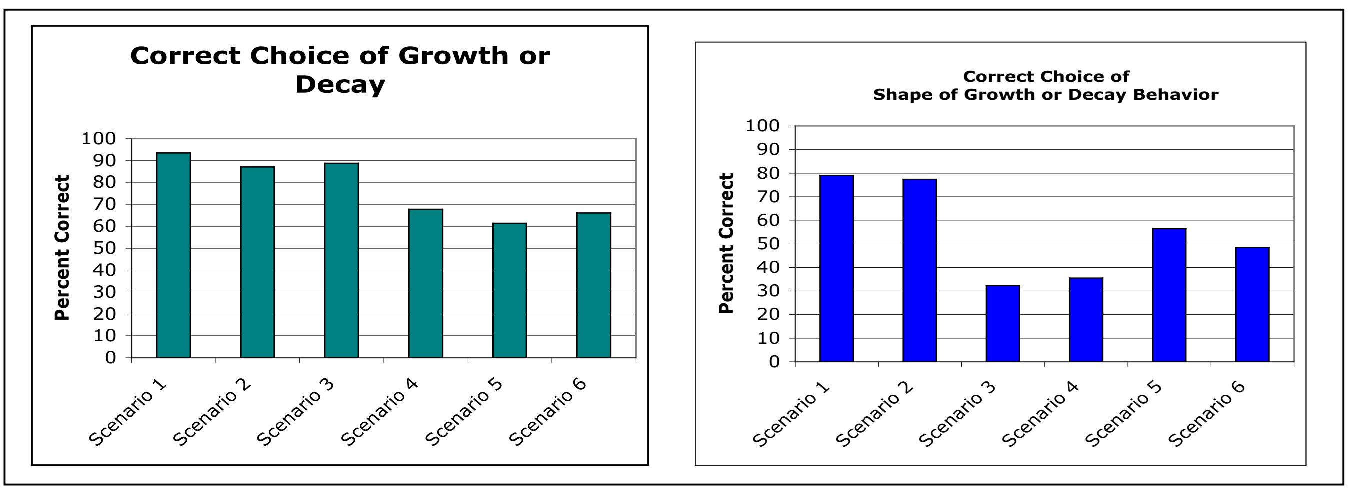


Figure 1: Percent of students who correctly identified the type of feedback present in the wolf-rabbit and in the births-population scenarios.



Figures 2a and 2b: The percent of students who correctly identified whether the scenario would produce growth or decay (2a), and who correctly identified the pattern of growth or decay (2b).

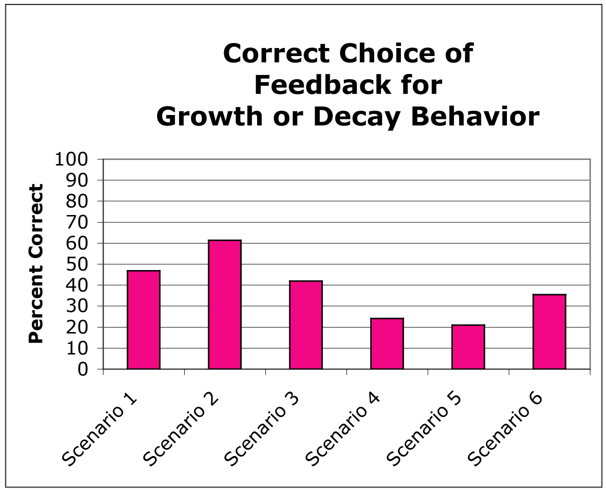


Figure 3: The percent of students who correctly identified the type of feedback present in each scenario.

Conclusions:

From the data shown in Figure 2a it appears that students are reasonably adept at determining, from a diagram, whether a structure will represent growth or decay if they have worked with the function type (linear or exponential) or are very familiar with the topic (population, showing growth fraction and death fraction). But when we combine different functions, linear and exponential, as in scenarios 4 and 5, students no longer have previous knowledge or intuition to fall back on. These required more original analysis. (Actually, one example of scenario 4, linear input and exponential output, had been demonstrated for the students when they started the drug model simulation activities a few days before this assessment was given. That probably accounts for the 5% improved performance on that question.) Surprisingly, 65% of students chose the correct growth/decay alternative for scenario 6, which was quite complicated. This may be due to the relative ease of reading the STELLA diagrams when full names can be used to describe the variables and parameters in the model. The students may have had some intuition about the simulation, since it involved population and a non-renewable food resource. (See footnote 3.)

Students did not perform as well when choosing the actual shape of the growth pattern or the decay pattern. With the exception of the linear and exponential scenarios (1 and 2) for which students have a great deal of experience, students struggled even to determine the shape of a population model behavior with exponential birth and death structures. For scenario 5, the exponential inflow so dominates the linear outflow, fairly obvious by just looking at the numbers, that it may have made this problem easier to analyze than scenario 4 where the inflow and outflow values are closer together from the beginning. (This is probably the sign that the scenario 5 values should have been modified slightly, more in line with the values of scenario 4.) Again, students did reasonably well on scenario 6, almost 50% correct, probably for the same reason described in the previous paragraph.

The feedback questions produced markedly different outcomes. With the definition of reinforcing and balancing feedback fresh in their minds, and two story scenarios that are clearly one type or the other, students responded with at least 90% accuracy with regard to the type of feedback present in each scenario. Those questions were very easy for the students. But when it came to trying to identify whether there was feedback present by observing the model diagram, students had a great deal of difficulty. Not even 50% of the students could determine correctly, that a linear structure would contain no feedback. Students were the most successful with the simplest feedback example, the sunflowers. That diagram was very closely related to the definition for reinforcing feedback. The very poor showing on the other scenarios seems to indicate that there was not enough instruction on feedback for students to be able to carry the feedback instruction given in the packet over to new situations. This task was just too difficult for students. They did not have enough experience to draw upon to analyze most of the scenarios. What these results indicate is that there should be formal instruction over multiple exercises to introduce and reinforce the concept of feedback.

It would be interesting to see how students performed on these scenarios if they had the mathematical equations to work from instead of the model diagrams. That would be another interesting experiment to try.

Bibliography:

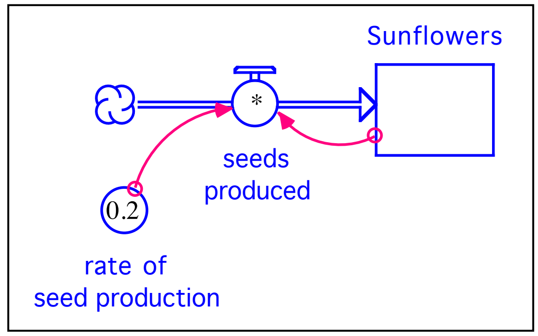
Linda Booth-Sweeney and John Sterman, Thinking about systems: student and teacher conceptions of natural and social systems. *System Dynamics Review* **23**(2-3): 285-311.J.W.Wiley.

Principles and Standards for School Mathematics, National Council of Teachers of Mathematics, 2000

Appendix A: The other five scenarios in the exercise.

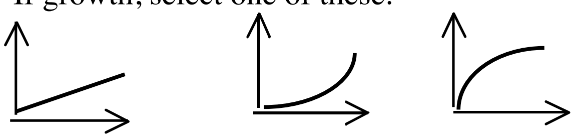
Scenario 2: Sunflowers

a. Will the amount of sunflowers (shown in the diagram at the right) grow or decay over time? _______________

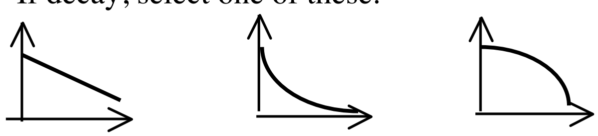


b. What will the pattern of growth or decay look like. Circle one graph below.

If growth, select one of these:



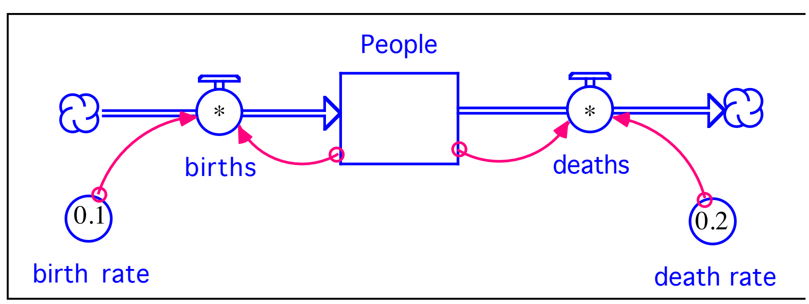
If decay, select one of these:



c. Is there feedback? _________ If so, what type of feedback is it? ______________ Explain

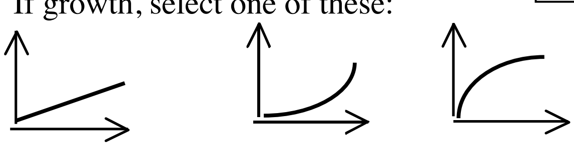
Scenario 3: Population

a. Will the population (shown in the diagram at the right) grow or decay over time? _______________

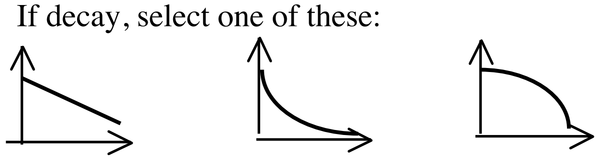


b. What will the pattern of growth or decay look like. Circle one graph below.

If growth, select one of these:



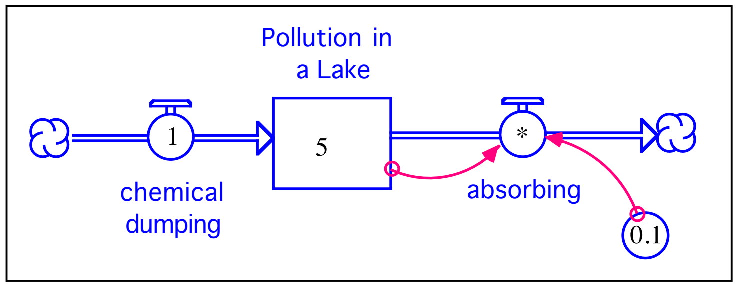
If decay, select one of these:



c. Is there feedback? ________ If so, what type of feedback is it? ______________ Explain

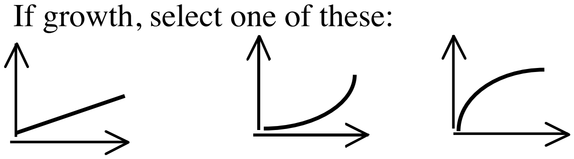
Scenario 4: Pollution

a. Will the pollution (shown in the diagram at the right) grow or decay over time? _______________

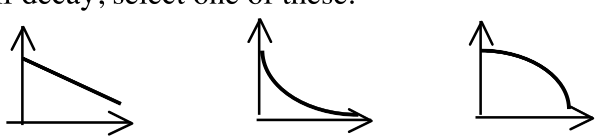


b. What will the pattern of growth or decay look like. Circle one graph below.

If growth, select one of these:

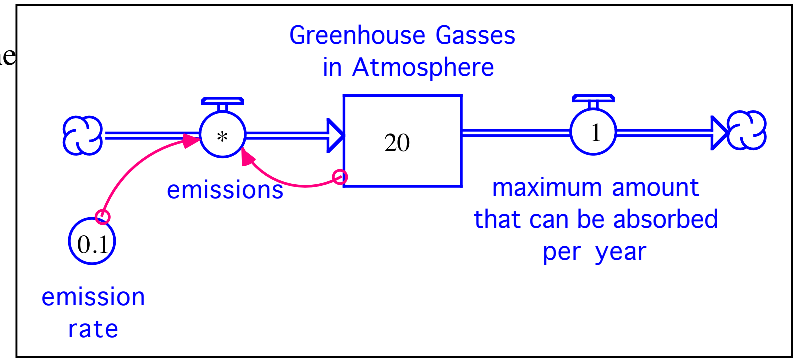


If decay, select one of these:



c. Is there feedback? ________ If so, what type of feedback is it? ______________ Explain

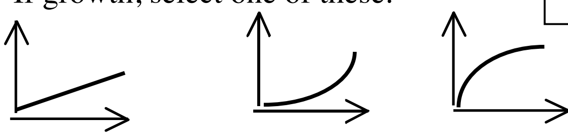
Scenario 5: Greenhouse gasses



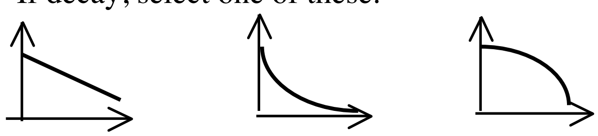
a. Will the greenhouse gasses (shown in the diagram at the right) grow or decay over time? _______________

b. What will the pattern of growth or decay look like. Circle one graph below.

If growth, select one of these:

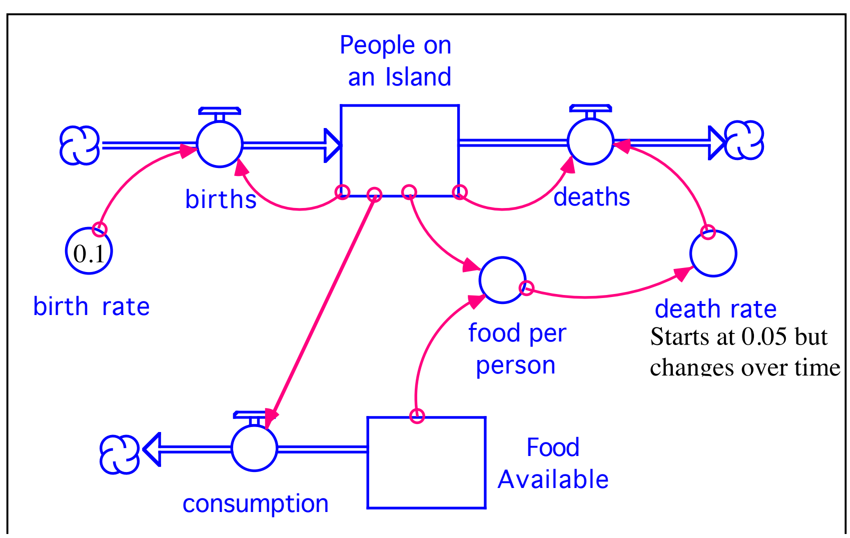


If decay, select one of these:



c. Is there feedback? _________ If so, what type of feedback is it? ______________ Explain

Scenario 6:



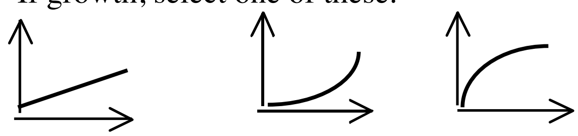
a. Will the population on the island (shown in the diagram at the right) grow or decay over time?

____

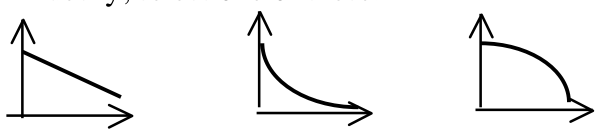
Notice that the food is not able to re-grow. It only gets consumed.

b. What will the pattern of growth or decay look like. Circle one graph below.[3]

If growth, select one of these:



If decay, select one of these:



c. Is there feedback? ________ If so, what type of feedback is it? _____________ Explain

[3] While there is no graph that represents the whole story for population change in this scenario (6), student answers that took into account the long-term behavior, eventual collapse for the population, were counted as correct answers. (It was not expected that most students would be able to analyze this scenario.)

Appendix B:

Pre-Exercise identifying feedback:
Identifying the correct type of feedback for the wolf-rabbit interaction.
Reinforcing (incorrect): 2 Balancing (correct): 58 Other (incorrect): 2

Identifying the correct type of feedback for the births-population interaction.
Reinforcing (correct): 56 Balancing (incorrect): 2 Other (incorrect): 4

Scenario 1: Making Candy
Identifying the scenario as growth (correct): 58 Decay (incorrect): 3 Other (incorrect): 1
Type of growth: linear (correct): 49 Exponential (incorrect): 6 Convergent(incorrect): 3
Type of decay: linear (incorrect): 0 Exponential (incorrect): 1 Collapse (incorrect): 2
Feedback type: None (correct): 29 Reinforcing (incorrect): 25 Balancing(incorrect): 2
Left blank (incorrect): 5 Other (incorrect): 1

Scenario 2: Sunflowers
Identifying the scenario as growth (correct): 54 Decay (incorrect): 5 Other (incorrect): 3
Type of growth: linear (incorrect): 5 Exponential (correct): 48 Convergent(incorrect): 1
Type of decay: linear (incorrect): 1 Exponential (incorrect): 3 Collapse (incorrect): 1
Feedback type: None (incorrect): 13 Reinforcing (correct): 38 Balancing(incorrect): 3
Left blank (incorrect): 7 Other (incorrect): 1

Scenario 3: Population
Identifying the scenario as growth (incorrect): 3 Decay (correct): 55 Other (incorrect): 4
Type of growth: linear (incorrect): 0 Exponential (incorrect): 2 Convergent(incorrect): 1
Type of decay: linear (incorrect): 8 Exponential (correct): 20 Collapse (incorrect): 27
Feedback type: None (incorrect): 15 Reinforcing (incorrect): 11 Balancing(correct): 26
Left blank (incorrect): 7 Other (incorrect): 3

Scenario 4: Pollution
Identifying the scenario as growth (correct): 42 Decay (incorrect): 14 Other (incorrect): 6
Type of growth: linear (incorrect): 4 Exponential (incorrect): 16 Convergent(correct): 22
Type of decay: linear (incorrect): 0 Exponential (incorrect): 7 Collapse (incorrect): 7
Feedback type: None (incorrect): 14 Reinforcing (incorrect): 22 Balancing(correct): 15
Left blank (incorrect): 7 Other (incorrect): 4

Scenario 5: Greenhouse gasses
Identifying the scenario as growth (correct): 38 Decay (incorrect): 15 Other (incorrect): 9
Type of growth: linear (incorrect): 1 Exponential (correct): 35 Convergent(incorrect): 2
Type of decay: linear (incorrect): 1 Exponential (incorrect): 4 Collapse (incorrect): 10
Feedback type: None (incorrect): 23 Reinforcing (correct): 13 Balancing(incorrect): 6
Left blank (incorrect): 13 Other (incorrect): 7

Scenario 6: (Not Named) Population and Food Depletion (See footnote 3)
Identifying the scenario as growth (incorrect): 8 Decay (correct): 41 Other (incorrect): 13
Type of growth: linear (incorrect): 0 Exponential (incorrect): 7 Convergent(incorrect): 7
Type of decay: linear (incorrect): 1 Exponential (incorrect): 10 Collapse (correct): 30
Feedback type: None (incorrect): 10 Reinforcing (incorrect): 9 Balancing(correct): 22
Left blank (incorrect): 16 Other (incorrect): 5